Assessment Schedule – 2005

Calculus: Integrate functions and solve problems by integration, differential equations or numerical methods (90636)

Evidence Statement

	Achievement Criteria	Q	Evidence	Code	Judgement	Sufficiency
Achievement	Integrate functions and solve problems by integration, differential equations or numerical methods.	1(a)	$\frac{3}{2}e^{2x-4}+c$	A1	Or equivalent.	**Achievement:** 4 × code A including at least 1 × code A1 and 1 × code A2.
		1(b)	$\frac{1}{3}\cot 3x + c$	A1	Or equivalent.	
		1(c)	$7x + 4\ \ln\lvert x\rvert + c$ **or** $7x + 4\ \ln\lvert kx\rvert$	A1	Or equivalent. Accept without | | sign.	
		2	$\frac{10}{3}\big(0 + 4(11.4 + 10.8 + 19.8 + 12.9) + 2(13.2 + 15.1 + 20.1) + 0\big)$ $= 1054\frac{2}{3}\ \text{m}^2$	A2	(Copying error possible MEI, calculation error, N) Or equivalent.	
		3	$\int \frac{1}{y}\text{d}y = \int (2x+1)\text{d}x$ $\ln y = x^2 + x + c$ $y = Ae^{(x^2+x)}$ $120 = Ae^{3.36}$ $y = 4.168e^{(x^2+x)}$ or $y = e^{(x^2+x+1.427)}$ or $\ln\lvert y\rvert = x^2 + x + 1.427$ or $\ln\lvert 0.2399y\rvert = x^2 + x$ or $y = 3.25e^{\frac{1}{4}(2x+1)^2}$	A2	Separation of variables must be shown; otherwise ^ NS and N at end. Or equivalent.	

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	Achievement Criteria	Q	Evidence	Code	Judgement	Sufficiency
Achievement with Merit	Find integrals and use integration to solve problems.	4(a)	$\frac{du}{dx} = 2$ $6(u+4) = 12x$ $\int 12x\sqrt{2x-4}\,dx$ $= \int 6(u+4)u^{\frac{1}{2}}\frac{1}{2}\,du$ $= \int 3u^{\frac{3}{2}} + 12u^{\frac{1}{2}}\,du$ $= \frac{6}{5}u^{\frac{5}{2}} + 8u^{\frac{3}{2}} + c$ $= \frac{6}{5}(2x-4)^{\frac{5}{2}} + 8(2x-4)^{\frac{3}{2}} + c$ **OR** Let $u = x - 2 \quad du = dx$ $x = u + 2$ So $\int = \int 12(u+2)(2u)^{\frac{1}{2}}\,du$ $= \int (12\sqrt{2}u^{\frac{3}{2}} + 24\sqrt{2}u^{\frac{1}{2}})\,du$ $= \frac{24\sqrt{2}}{5}u^{\frac{5}{2}} + 16\sqrt{2}u^{\frac{3}{2}} + c$ $= \frac{24\sqrt{2}}{5}(x-2)^{\frac{5}{2}} + 16\sqrt{2}(x-2)^{\frac{3}{2}} + c$ **OR** $u = \sqrt{2x-4}$ $u^2 = 2x - 4$ $2u\,du = 2\,dx$ $u\,du = dx$ $x = \frac{u^2+4}{2}$ $\int = \int 6(u^2+4).u.u\,du$ $= \int (6u^4 + 24u)\,du$ $= \frac{6}{5}u^5 + 8u^3 + c$ $= \frac{6}{5}(2x-4)^{\frac{5}{2}} + 8(2x-4)^{\frac{3}{2}} + c$ **OR** $4x(2x-4)^{\frac{3}{2}} - \frac{4}{5}(2x-4)^{\frac{5}{2}} + c$	A1 M	Or equivalent. Or equivalent.	**Achievement with Merit:** **EITHER** As for Achievement **plus** **3** × code M **OR** 4 × code M.
		4(b)	$4\int(\cos 4x + \cos 2x)\,dx$ $= \sin 4x + 2\sin 2x + C$	A1, M	Or equivalent.	

	Achievement Criteria	Q	Evidence	Code	Judgement	Sufficiency
		5	$\frac{dN}{dt} = kN$ $\int \frac{1}{N} dN = \int k\, dt$ $\ln\lvert KN\rvert = kt$ $N = Ae^{kt}$ $120 = Ae^{20k}$ $85 = Ae^{32k}$ $k = -\ 0.0287367$ $120 = Ae^{-0.0287367 \times 20}$ $A = 213.199$ $30 = 213.199e^{-0.0287367t}$ $t = 68.2$ Accept 68 or 69 days.	A2, M	Evidence of appropriate integration required. Don't penalise premature rounding. Or equivalent.	DE must be shown otherwise ^ NS; possible to obtain code A2 if pursue solution to the end.
		6	Volume $= \pi\int_3^7 (y-3)\, dy$ $= \pi\left[\frac{y^2}{2} - 3y\right]_3^7$ $= 8\pi$ $= 25.13\ m^3$	A1 A1, A2	Accept any valid method. Or equivalent. Do **not** allow $y+3$ or $\pi\int_3^7 (3-y)\, dy$	Correct integral, even if limits incorrect, replacement generates code A1. The integral must be shown – otherwise ^ NS and N.

	Achievement Criteria	Q	Evidence	Code	Judgement	Sufficiency
Achievement with Excellence	Use a variety of integration techniques to solve problems.	7	Point of intersection of $y^2 = 6x$ and $y^2 = 36 - 9x$: $6x = 36 - 9x$ $x = \frac{36}{15}$ or 2.4 Area between the three parabolas: $= \int_0^{2.4} \sqrt{6}x^{0.5} - \frac{1}{4}x(x-4)\,dx + \int_{2.4}^{4} (36-9x)^{0.5} - \frac{1}{4}x(x-4)\,dx$ $= \left[\frac{2\sqrt{6}}{3}x^{\frac{3}{2}} - \frac{x^3}{12} + \frac{x^2}{2}\right]_0^{2.4} + \left[-\frac{2}{27}(36-9x)^{\frac{3}{2}} - \frac{x^3}{12} + \frac{x^2}{2}\right]_{2.4}^{4}$ $= 7.7996 + 4.9864$ $= 12.79$ to 2 dp. $6.072 + 4.048 + 2.666$ $= 12.79$ to 2 dp.	A1 A2, M, E	Accept any valid method. Accept minor arithmetic error. Or equivalent.	**Achievement with Excellence:** as for Merit **plus** code E. The integrals must be shown, and be correct, for code A1 replacement; otherwise ^ NS and N at end. All calculations must also be correct for codes E, M or A2.

Judgement Statement

Achievement	Achievement with Merit	Achievement with Excellence
Integrate functions and solve problems by integration, differential equations or numerical methods. 4 × A *including* at least 1 × A1 *and* 1 × A2.	Find integrals and use integration to solve problems. Achievement *plus* 3 × M ***or*** 4 × M	Use a variety of integration techniques to solve problems(s). Merit *plus* 1 × E